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ASSESSING PREDICTABILITY OF HYDROLOGICAL PROCESSES (ON THE EXAMPLE OF FROZEN SOIL WATER CONTENT DYNAMICS)

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Abstract. A method has been developed for assessing the limits of predictability of the frozen soil water content (according to observations at the Nizhnedevitskaya water balance station). The method is based on the analysis of the convergence of a given probabilistic measure (the variance of the calculated soil water content at a given date) to its stable value. The soil water content was simulated by the physically based model of heat and water transfer in a frozen soil column during a autumn-winter seasons. To assess variability of the modelled soil water content at a given date, the boundary meteorological conditions for the autumn-winter period were simulated by the Monte Carlo procedure using a stochastic weather generator. The initial conditions were assigned as the constant soil temperature and soil moisture values over the 1-meter soil column. The predictability of the soil water content in the one-meter layer of the studied soils has occurred to be about 1.5 months; it means that for the forest-steppe conditions, the soil water content before the beginning of soil freezing cannot serve as an indicator of soil water content before spring. Numerical experiments have shown that the soil water content predictability: (1) grows with an increase in the thickness of the considered soil layer and its depth; (2) decreases for coarser soils as compared to finely dispersed soils; (3) is more sensitive to changes in the soil texture than to changes in the climatic norms of precipitation and air temperature.

ОЦЕНКА ПРЕДСКАЗУЕМОСТИ ГИДРОЛОГИЧЕСКИХ ПРОЦЕССОВ (НА ПРИМЕРЕ ДИНАМИКИ ВЛАЖНОСТИ МЕРЗЛОЙ ПОЧВЫ)

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Аннотация. Разработана методика оценки пределов предсказуемости влажности мерзлой почвы (по данным наблюдений на Нижнедевицкой воднобалансовой станции). Методика основана на анализе сходимости заданной вероятностной меры (дисперсии расчетной влажности почвы на заданную дату) к ее устойчивому значению. Влагосодержание почвы рассчитывалось с помощью физико-математической модели вертикального тепло- и влагопереноса в почвенной колонке в осенне-зимний период. Чтобы оценить вариацию величины влажности почвы, рассчитанной на заданную дату, граничные метеорологические условия в течение осеннее-зимнего периода моделировались с помощью процедуры Монте-Карло с использованием стохастического генератора погоды. Начальные условия задавались в виде профилей постоянных значений температуры и влажности почвы по глубине. Расчетная оценка предсказуемости влажности почвы в метровом слое исследуемых почв составила около 1,5 месяцев, то есть для условий лесостепи осенняя влажность почвы не может служить индикатором ее увлажнения перед началом весеннего снеготаяния. Численные эксперименты показали, что предсказуемость влажности почвы: 1) растет с увеличением толщины рассматриваемого слоя почвы и глубины этого слоя; 2) уменьшается для почв легкого механического состава по сравнению с почвами тяжелого состава; 3) более чувствительна к изменению текстуры почвы, чем

к изменению климатических норм осадков и температуры воздуха.

Keywords: hydrological processes; predictability; physically-based model; weather generator; soil moisture dynamics; frozen soil

Ключевые слова: гидрологические процессы; предсказуемость; физико-математическая модель; генератор погоды; динамика почвенной влаги; мерзлая почва

Introduction

The study of the predictability of natural processes is a traditional task for many geophysical sciences. In meteorology, for example, such studies are based on the concepts of two types of predictability developed in the classical works of E. Lorenz [Lorenz, 1975]: predictability of the first kind, due to the internal instability of atmospheric processes, their high sensitivity to small perturbations of the initial conditions and predictability of the second kind, associated with the variability of external influences with respect to the atmosphere (ocean, land surface, etc.). The study of the physical mechanisms and factors of predictability, the determination of its limits depending on the space-time scales of the processes under study, the identification of potentially predictable structures – the listed and other problems of scientific meteorology are solving on the basis of the concept of predictability as a problem of mathematical physics developed already half a century ago (see, for example, [Dymnikov, 2007]).

In hydrology, the concept of predictability over many years is associated, as a rule, with the possibility of constructing a forecast method that meets the specified criteria of accuracy and lead time (see, for example, [Shukla et al., 2013]). This understanding of predictability is close to the concept of “effective predictability” [Douville, 2010] or “forecastability” accepted in meteorology, which depends, inter alia, on subjective factors, including the experience and methodological preferences of a forecast developers, the characteristics of the existing observation network, etc. Interest in the analysis of predictability as an objective property of the hydrological system has arisen in recent years and is primarily associated with studies of the macroscale variability of soil moisture and snow cover characteristics aimed at deepening understanding of the contribution of low-frequency variability of these characteristics to the dynamics of the climate system[1]. In addition, publications have appeared in which factors influencing the predictability of hydrological processes on a river basin scale are investigated, such as: spatial averaging of processes [Blöschl, Sivapalan, 1995], nonlinearity of hydrological systems [Zehe et al., 2007], hydraulic properties of the basin [Kumar, 2011]. There is reason to believe that the construction of a conceptual framework for analyzing the predictability of hydrological systems is becoming one of the key scientific problems in hydrology [Blöschl, 2006].

The purpose of this study is to develop a methodological approach to assessing the predictability limits (potential predictability) of hydrological processes using the example of moisture transfer processes and the formation of soil moisture content in a frozen soil. The moisture content of frozen soil is the main factor in the snowmelt runoff losses during the formation of spring floods (freshets) in the cold regions’ river basins, and the reliability of its assessment significantly affects the accuracy of forecasts of spring floods using the existing operational techniques. At the same time, measurements of soil moisture in winter are mostly not performed in Russia (there are only point measurement data at the experimental sites of water balance stations and small experimental basins), therefore, to assess the moisture content of frozen soil before the onset of snow melting, existing forecast methods use the so-called indices of preliminary

[1] Report of a Workshop on Predictability & Limits-To-Prediction in Hydrologic Systems. Washington, D.C., National Academy Press, 2002. 138 p.

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moisture content[2]. Such indices are, as a rule, data on the soil moisture content or on the low-flow runoff for the previous autumn period. In other words, the hypothesis is implicitly accepted that the moisture content of frozen soil before the onset of snow melting depends to a greater extent on the autumn soil moisture than on moisture transfer processes in winter. The validity of this hypothesis depends on many factors, primarily on the weather conditions during the winter, soil properties and vegetation cover. A quantitative estimate of the time period during which the influence of the previous (autumn) conditions persists, depending on the listed factors, constitutes the essence of the approach proposed in the article and can serve as a methodological basis for testing this hypothesis.

Methods

Approach to assessing the predictability limit of a hydrological system

Consider a stochastical dynamical system described by the equation [Dymnikov, 2007]:

$$\frac{dW}{dt} = B(W) + \varepsilon(t) \tag{1}$$

$$W\big|_{t=0} = W_0 \tag{2}$$

where $W = W(t)$ is a state variable;

$B(W)$ is a dynamics operator;

$\varepsilon(t)$ – delta-correlated Gaussian random process $\left\langle \varepsilon_i(t) \times \varepsilon_j(t') \right\rangle = 2d_{ij}\delta(t-t')$, $d_{ij} > 0$;

$\langle \ \rangle$ – averaging operator.

In hydrology, the dynamic-stochastic equation (1) is used to describe the soil moisture dynamics, interannual changes in river runoff, fluctuations in the lake level, and the dynamics of the volume of mountain glaciers (see, for example, [Demchenko, Kislov, 2010]. It was shown [Dymnikov, 2007] that, under the condition $d_{ij} \equiv d$, equation (1) can be written in the form of the Fokker-Planck equation for the probability density function $\rho(W)$:

$$\frac{d\rho}{dt} + div\big(B(W)\rho\big) = d\Delta\rho \tag{3}$$

$$\rho \ge 0, \int \rho dW = 1 \tag{4}$$

The initial condition for the equation (3) is written as:

$$\rho\big|_{t=0} = \delta(W - W_0) \tag{5}$$

It was also shown [Dymnikov, 2007] that over time, the probability density function $\rho(W)$ of the system state will converge to a stationary solution $\bar{\rho}$, and information about the initial condition will be lost by the system. The time interval during which the system retains information about its initial state can be set by the time of convergence of a given probabilistic measure to its stable value and determines the potential predictability of the system or the physically determined limit of its predictability.

For a complex nonlinear system, the implementation of the described approach is possible through experiments with a numerical dynamical-stochastic model with random inputs, which describes the dynamics of the system states. This possibility will be illustrated below using the example of a dynamic-stochastic model of moisture transfer in a frozen soil.

Dynamic-stochastic model of water transfer in a frozen soil

The structure of the developed dynamic-stochastic model combines two components: (1) a physically based model of vertical moisture transfer in a frozen soil, taking into account the accumulation and melting of snow cover on its surface, and (2) a stochastic weather generator.

The physically based model is described in [Gelfan, 2006] and based on solving a system of nonlinear equations of heat and moisture transfer describing the hydrothermal regime of the soil during its freezing, thawing and infiltration of melt water, taking into account the phase water-ice transformation, the influence of over-cooled moisture. The system of equations and methods of their numerical integration, the results of model tests based on the data of laboratory and field experiments of the Hydrophysical Laboratory of the State Hydrological Institute and field observations at the

[2] Guide to Hydrological Forecasts. Issue 1. Long-term forecasts of the elements of the water regime of rivers and reservoirs. Leningrad, Publ. Gidrometeoizdat, 1989. 358 p. (In Russian).

Nizhnedevitskaya water balance station are presented in detail in [Gelfan, 2006]. The used stochastic weather generator, which allows one to reproduce by the Monte Carlo method long-term artificial sequences of meteorological variables with daily resolution at a given point (without taking into account the spatial relationships between meteorological variables) is described in [Gelfan, 2007]. The paper describes the results of calibration and testing of the weather generator based on meteorological measurements on the territory of the forest-steppe zone of the European Russia.

Numerical experiments design

Numerical experiments to assess the potential predictability of soil moisture content in the autumn-winter period were carried out using observational data at the Nizhnedevitskaya water balance station and were organized as follows.

Calculations were made for the period from November 1 to March 31. Stochastic weather generator was developed for this season on the basis of the meteorological observations from 1969 till 2014. The initial conditions for solving the equations of heat and moisture transfer were set as the constant total (liquid water + ice) soil moisture content and the constant soil temperature over the depth of a one meter soil column. An ensemble of 1 000 random realizations of time series of meteorological variables (air temperature, precipitation and air humidity) with daily resolution for the period from November 1 to March 31 was modeled using a stochastic weather generator. These variables were set as boundary conditions for the calculated equations of heat and moisture transfer in the absence of snow. If there is a snow cover on the soil surface (its characteristics were calculated from the generated meteorological "inputs" using a single-layer model [Gelfan, 2006]), the boundary conditions were set in the form of heat and moisture fluxes at the snow-soil boundary.

Using a dynamic-stochastic model, an ensemble of N=1 000 trajectories of changes in daily values of soil moisture $W_{ijk}=W_k(z_i,t_j)$ and temperature $T_{ijk}=T_k(z_i,t_j)$ at different depths was calculated from November 1 to March 31 (here k is the number of the calculated trajectory, k=1,2,…,1000; z_i=[10 (i-1); 10i] [cm] is i- th calculated 10-cm layer along the depth of a meter soil column from its surface; i=1,2,…,10; t_j=[(j-1); j], [days], is j-th daily time interval from the beginning of calculations j=1,2,…,151).

Each of the 1000 trajectories was calculated under given (the same for all calculations) initial conditions and random, simulated by the Monte Carlo method, boundary conditions.

For each calculated step t_j and soil layer z_i, the mean square deviation $\sigma_{ij}(W)$ of soil moisture was calculated for the ensemble N=1 000 trajectories:

$$\sigma_{ij}(W)=\sqrt{\sum_{k=1}^{N}\left(W_{ijk}-\overline{W}_{ij}\right)^2\times(N-1)^{-1}} \qquad (6)$$

where $\overline{W}_{ij}=N^{-1}\sum_{k=1}^{N}W_{ijk}$ is an average over the ensemble of trajectories value of soil moisture in the layer z_i and at the time step t_j.

The value $\sigma_{ij}(W)$ was specified as a probabilistic measure, the time of convergence of which to a stable value was assigned as the potential predictability of soil moisture.

The limit of predictability $\Delta_i(W)$ of soil moisture (potential predictability) was estimated from the condition:

$$\Delta_i=\min(j):\left|\frac{\sigma_{ij}-\sigma_{i(j-1)}}{\sigma_{ij}}\right|\le\alpha\,,\ j=1,2\ldots,151 \qquad (7)$$

where α is the specified threshold value (taken equal to 0.01).

Thus, the predictability limit of the process is determined by the time period during which the variance of the scatter of trajectories calculated for each time step becomes a constant value determined by external meteorological forcing (winter weather conditions), and the effect of initial conditions (pre-winter moisture and soil temperature) becomes insignificant.

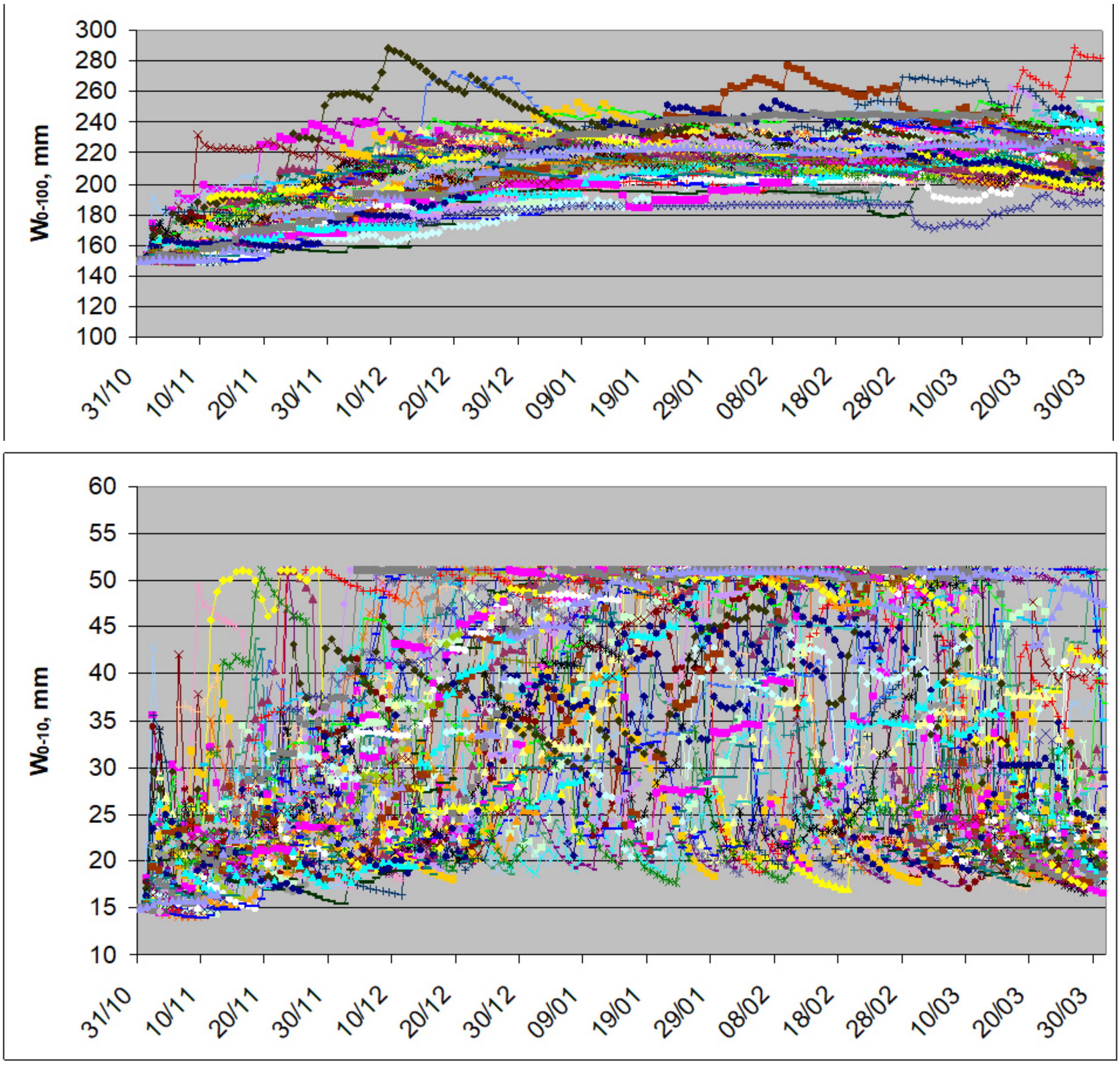

Figure 1. Ensemble of trajectories of the total moisture content (mm) of the meter (top) and upper 10-cm (bottom) soil layers of freezing soil, calculated under the same initial conditions (*W(z,0)*=0.15; *T(z,0)*=3°C) and the Monte Carlo simulated boundary meteorological conditions

Рисунок 1. Ансамбль траекторий изменения влагосодержания (мм) метрового слоя (вверху) и верхнего 10-см слоя промерзающей почвы, рассчитанных при начальных условиях (*W(z,0)*=0,15; *T(z,0)*=3°C) и граничных метеорологических условиях, сгенерированных методом Монте Карло

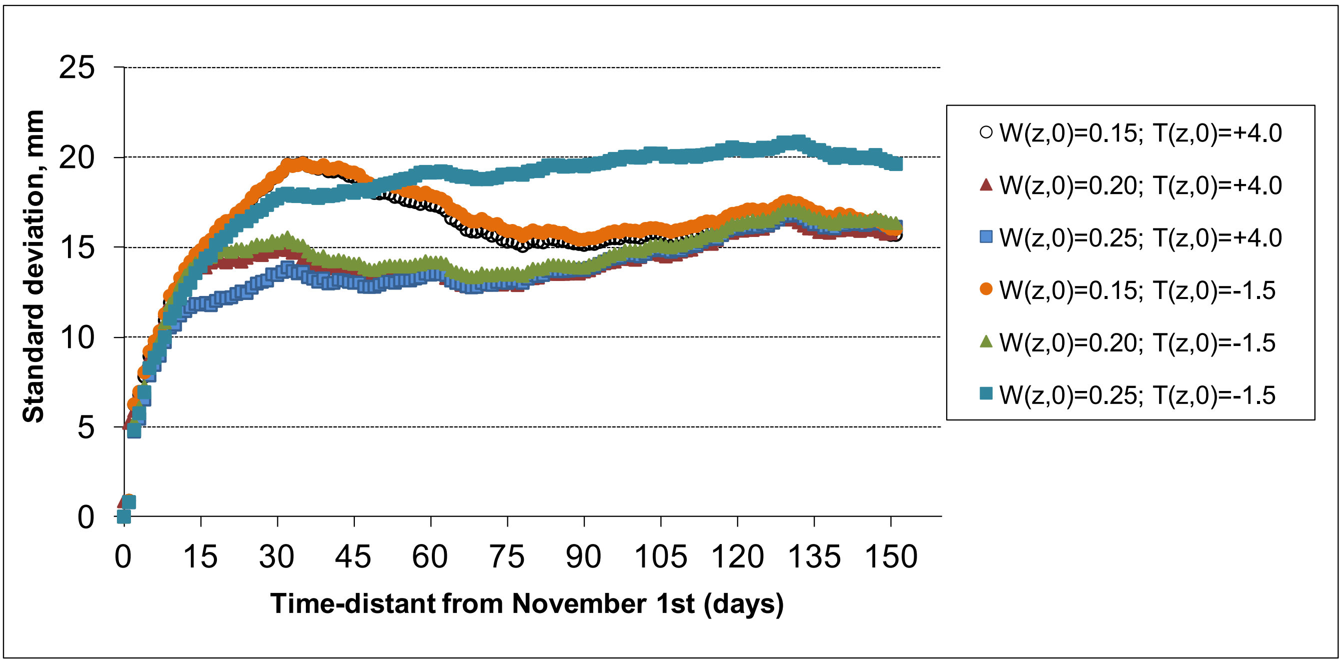

Figure 2. Change in the mean square deviation (over the ensemble of 1 000 trajectories) of the freezing soil moisture content (meter soil layer) with time-distance from the initial date (November 1). Each line relates to the specific initial conditions

Рисунок 2. Изменения среднеквадратического отклонения (по ансамблю 1000 траекторий) значений влагосодержания метрового слоя почвы с удалением от начальной расчетной даты (1 ноября). Каждая линия на графике относится к конкретным начальным условиям

Results and discussion

The calculated trajectories of changes in the total moisture content are shown as an example in figure 1.

Figure 2 shows how the value $\sigma_{ij}(W)$ changes with the time-distance from the initial date of calculation (November 1) under different initial conditions (the total moisture content of a meter soil layer of freezing soil is shown as an example).

Figure 2 shows that the predictability limit of the total moisture content of the soil column, calculated by formula (4) at α=1%, varies in a rather narrow range, namely from 31 to 42 days, with the given significant changes in the initial conditions ($W(z,0)$= 0.15÷0.25; $T(z,0)$=-1.5÷+4°C).

In general, the obtained estimates of the predictability of the moisture reserve for individual 10-cm layers of the freezing soil column showed that predictability increases with increasing layer depth: for the upper layer, predictability is about one month, for the lower layer 90-100 cm it reaches 3 months (figure 3).

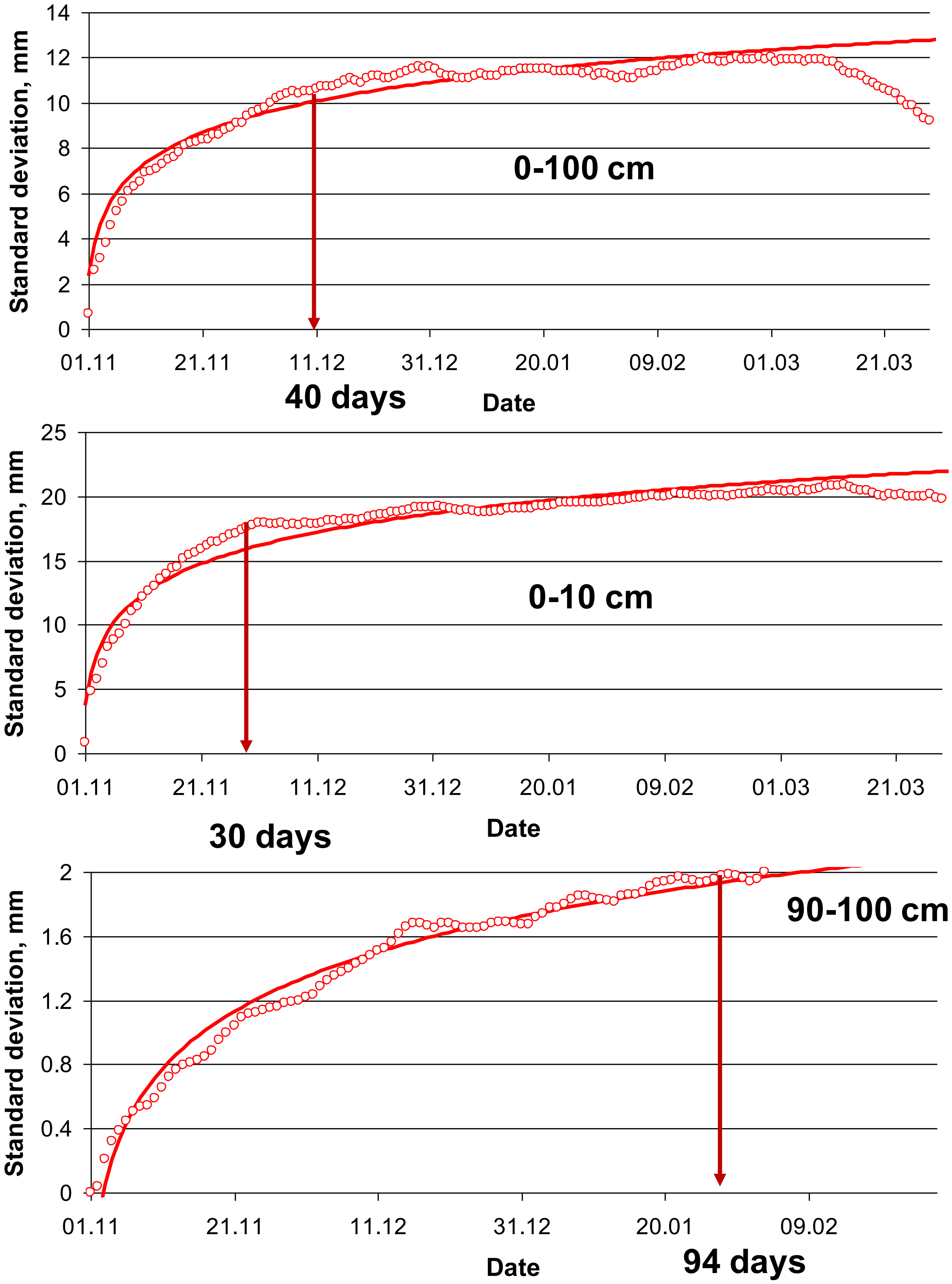

Figure 3. Change in the standard deviation and assessment of the predictability of the moisture content in the meter soil layer (top figure), moisture content in the 0-10 cm layer (middle figure) and moisture content in the 90-100 cm layer (bottom figure)

Рисунок 3. Изменения среднеквадратического отклонения и оценки предсказуемости влагосодержания метрового слоя почвы (вверху), верхнего 10-см слоя почвы (средний рисунок) и слоя 90-100 см (нижний рисунок)

Two series of numerical experiments were carried out to assess the sensitivity of the calculated predictability limit of the 1-meter soil moisture content: (1) to changes in soil texture, which affects the hydraulic and thermal parameters of the equations of heat and moisture transfer, and also (2) to changes in climatic norms of air temperature and precipitation specified as parameters of the stochastic weather generator. The results of numerical experiments are shown in figures 4, 5. The experiments have shown that the predictability of the 1-meter soil moisture content increases for soils with a heavier texture, reaching 51 days for heavy loam (figure 4). To assess sensitivity of predictability to the climatic norms (mean annual values), four numerical experiments were carried out. First, the air temperature climatic norm, T_a, as the weather generator parameter was increased by 2°C with an unchanged precipitation climatic norm (hereafter, "Scenarios 1"). Second, T_a was increased by 1°C with an unchanged precipitation norm ("Scenarios 2"). Third, the annual precipitation P was increased by 10% at an unchanged air temperature norm ("Scenarios 3") and, fourth, P increased by 20% at an unchanged air temperature norm ("Scenarios 4"). The calculation results are shown in figure 5. One can see that the assigned changes in climatic characteristics did not have a noticeable effect on the predictability limit of soil moisture content.

Thus, an important property of the system under consideration is that its predictability depends on the internal properties of the system to a greater extent than on the characteristics of the forcing process. A similar result for a linear dynamical-stochastic system was obtained analytically in [Demchenko, Kislov, 2010].

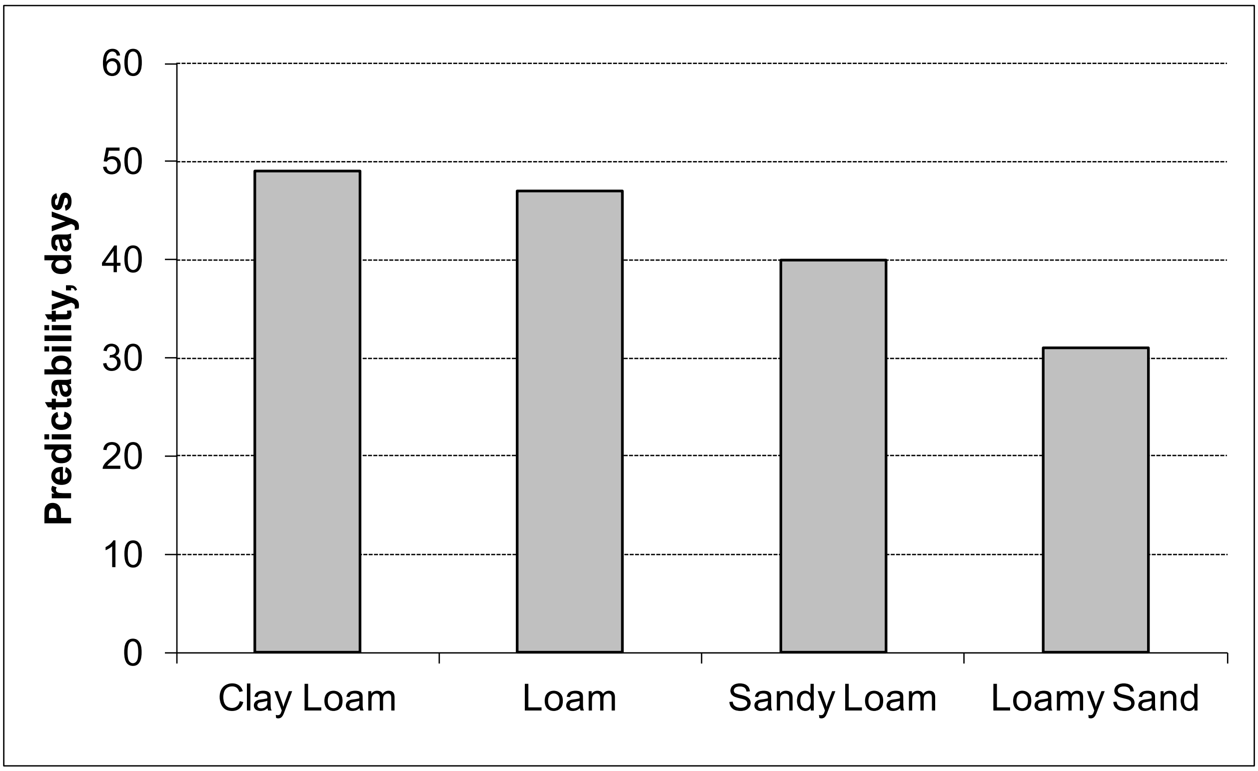

Figure 4. Predictability of moisture content of a meter layer of freezing soil, depending on soil texture
Рисунок 4. Предсказуемость влагозапаса метрового слоя промерзающей почвы разного механического состава

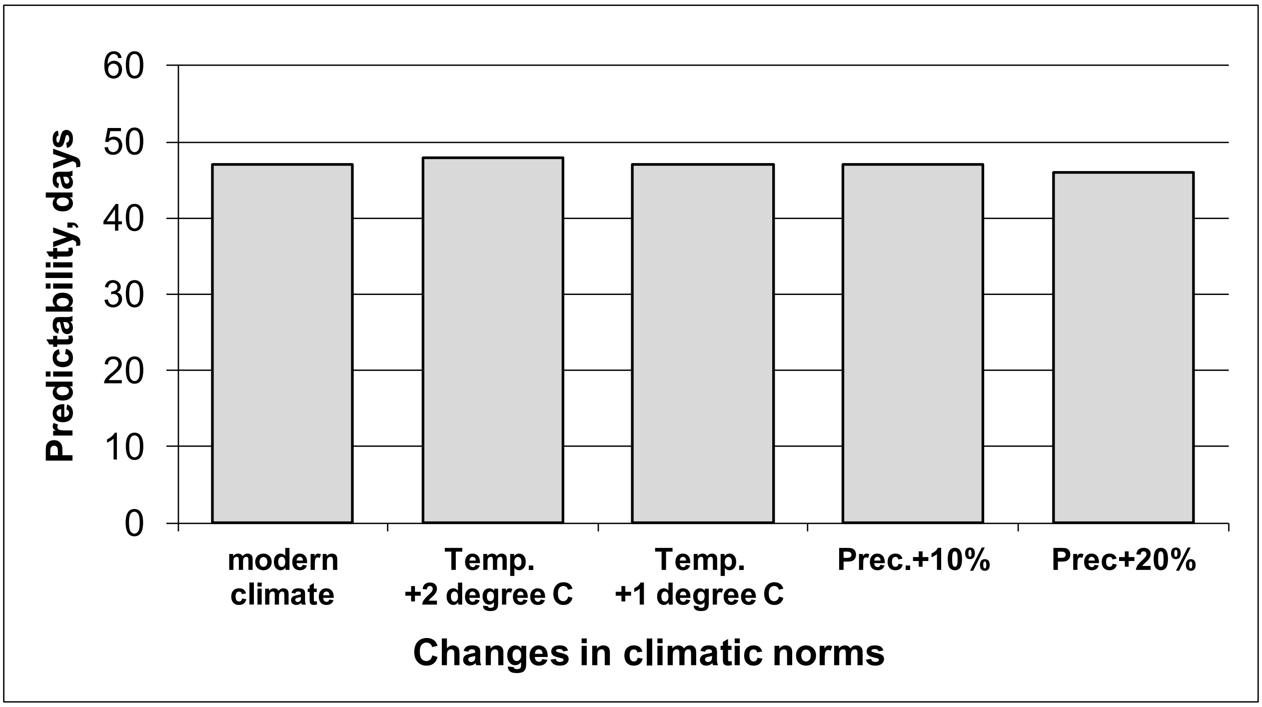

Figure 5. Predictability of moisture content in a meter layer of freezing soil under different scenarios of changes in climatic norms of air temperature and precipitation

Рисунок 5. Предсказуемость влагозапаса метрового слоя промерзающей почвы при 4-х сценариях изменений климатических норм температуры воздуха и осадков

Conclusions

The paper proposes a method for estimating the predictability limits of a hydrological system based on a dynamic-stochastic model with random inputs. The method is based on calculating the time of convergence of the variance of the state of the system to its stable value. The method was applied to assess the predictability of moisture content in a freezing soil. Using numerical experiments with the developed model, the following is shown.

1. The predictability of the moisture content of the freezing soil grows with an increase in the thickness and depth of the considered soil layer.

2. In soils of light texture, the predictability of moisture content is less than in heavier soils.

3. The predictability of soil moisture content depends on the hydraulic properties of the soil to a much greater extent than on the parameters of the meteorological forcing (climatic norms of precipitation and air temperature).

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